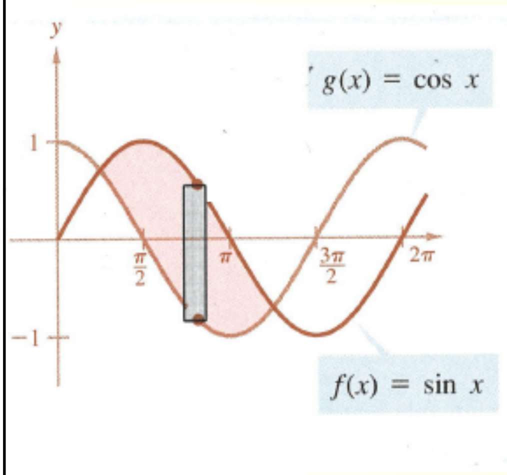
Calculus Warm Up # 9-5

The sine and cosine curves intersect infinite times.
Find the area of one of these regions.



HW Questions: p. 497

23. $\int \frac{1}{1 - \cos x}\,dx$

25. $\int \frac{2t - 1}{t^2 + 4}\,dt$ $\rightarrow \int \frac{2t}{t^2+4}\,dt - \int \frac{1}{t^2+4}\,dt$

27. $\int \frac{3}{t^2 + 1}\,dt$

29. $\displaystyle\int \frac{1}{x\sqrt{x^2-4}}\,dx$

31. $\displaystyle\int \frac{-1}{\sqrt{1-(2t-1)^2}}\,dt$

$u = x \quad a = 2$

37) $\displaystyle\int \frac{\sec^2 x}{4+\tan^2 x}\,dx$

$u = \tan x$

$du = \sec^2 x\, dx$

$\displaystyle\int \frac{1}{\underset{a=2}{2^2} + \underset{u}{(\tan x)^2}} \cdot \underbrace{\sec^2 x\, dx}_{du}$

47) $\displaystyle\int \frac{3}{\sqrt{6x-x^2}}\,dx$

$-(x^2-6x+\underline{9})+9$

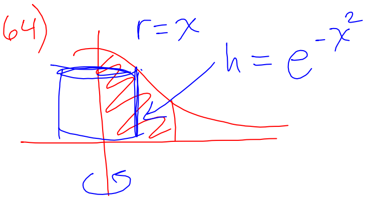
$-(x-3)^2+9$

$3^2-(x-3)^2$

$\displaystyle 3\int \frac{1}{\sqrt{3^2-(x-3)^2}}\,dx$

49) $\int \frac{4}{4\left[\left(x+\frac{1}{2}\right)^2+4^2\right]}\,dx$

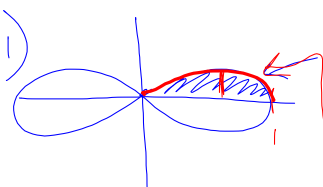
64)



Shells
$2\pi r \cdot h$

$V = 2\pi \int_0^1 x e^{-x^2}\,dx$

61)



$4\int_0^1 x\sqrt{1-x^2}\,dx$

$y^2 = x^2(1-x^2)$

$y = x\sqrt{1-x^2}$

66) a) $\frac{2}{\pi}$

b) $\frac{1}{3}\arctan 3$

68) $\frac{8\pi}{3}\left(10^{3/2}-1\right)$

≈ 256.5

9.2 Integration by Parts

$$\int \underbrace{x}\,\underbrace{e^x\,dx}$$

Too many "variable parts" that don't match.

We need a new tool!

Based on the Product Rule:

$$\frac{d}{dx}[u\,v] = u\cdot\frac{dv}{dx} + v\,\frac{du}{dx}$$

$$\int \frac{d}{dx}[u\,v] = \int u\cdot\frac{dv}{\cancel{dx}}\cancel{dx} + \int v\,\frac{du}{\cancel{dx}}\cancel{dx}$$

$$uv = \int u\,dv + \int v\,du$$

Rearrange: $\boxed{\int u\,dv = uv - \int v\,du}$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Name the parts u and dv

$$\int x e^x dx$$

$u = x \qquad dv = e^x dx$

Then find du & v:

$du = dx \qquad v = \int e^x dx$

$v = e^x$

$$\int x e^x dx = xe^x - \int e^x dx$$

$$= xe^x - e^x + C$$

$$\int x^2 \ln x\, dx$$

$\star$ $\int \ln x\, dx$ not a basic formula so

$u = \ln x \qquad dv = x^2 dx$

$du = \frac{1}{x} dx \qquad v = \frac{x^3}{3}$

$$\int u\,dv = uv - \int v\,du$$

$$= (\ln x)\frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} dx$$

$$= \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2 dx$$

$$\frac{x^3}{3}\ln x - \frac{1}{3} \cdot \frac{x^3}{3} + C$$

$$\frac{x^3}{3}\ln x - \frac{x^3}{9} + C$$

$\int \arcsin x \, dx$ $\rightarrow$ No formula so:

$u = \arcsin x \qquad dv = 1\,dx$

$du = \frac{1}{\sqrt{1-x^2}} dx \qquad v = x$

$$\int \arcsin x \, dx = x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx$$

$u = 1 - x^2$

$du = -2x\,dx$

$$= x \arcsin x + \frac{1}{2} \int (1-x^2)^{-1/2} (-2x\,dx)$$

$$= x \arcsin x + \frac{1}{2} \left[\frac{2u^{1/2}}{1} \right] + C$$

$$= x \arcsin x + \sqrt{1-x^2} + C$$

$\int x^2 \sin x \, dx$

Since $\frac{d}{dx} x^2$ gets simpler,

let $u = x^2 \qquad dv = \sin x \, dx$

$du = 2x\,dx \qquad v = -\cos x$

$$= x^2(-\cos x) - \int (-\cos x)(2x\,dx)$$

$u = 2x \qquad dv = \cos x$

$du = 2\,dx \qquad v = \sin x$

$$= -x^2 \cos x + \int 2x \cos x \, dx$$

again!

$$= -x^2 \cos x + 2x \sin x - \int \sin x (2\,dx)$$

$$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

$$\int x^2 \sin x \, dx$$

Table Method

signs	$u \downarrow$	$dv \uparrow$
$+$	x^2	$\sin x$
$-$	$2x$	$-\cos x$
$+$	2	$-\sin x$
$-$	0	$\cos x + C$

Answer

$$= +x^2(-\cos x) - 2x(-\sin x) + 2\cos x + C$$

$$= -x^2 \cos x + 2x \sin x + 2\cos x + C$$

$$\int x^2 e^{2x} \, dx$$

signs	$u \downarrow$	$dv \uparrow$
$+$	x^2	e^{2x}
$-$	$2x$	$\frac{1}{2}e^{2x}$
$+$	2	$\frac{1}{4}e^{2x}$
$-$	0	$\frac{1}{8}e^{2x}$

$+C$

$$\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$$

Classwork: Purple WS

Answers

1) $\operatorname{arcsec}|2x| + C$

2) $\arcsin e^x + C$

3) $2\arctan(\sqrt{x}) + C$

4) $\frac{\pi}{2}$

5) $\arcsin\left(\frac{x+2}{2}\right) + C$

6) $\frac{\pi}{4}$

7) $\ln|x^2+6x+13| - 3\arctan\left(\frac{x+3}{2}\right) + C$

8) $-\cot x - \csc x + C$

9) $\frac{1}{2}\arctan(x^2+1) + C$

10) :(We need a new tool.

$du = e^x dx$

1)

$$\int \frac{1 \cdot 2}{\underbrace{2x}_{u} \sqrt{4x^2-1}}\,dx$$

$u = 2x$

$du = 2\,dx$

$$3)\ 2\int \frac{1}{2\sqrt{x}\,(1+x)}\,dx$$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$$2\int \frac{1}{1^2+(\sqrt{x})^2} \cdot \underbrace{\frac{1}{2\sqrt{x}}\,dx}_{du}$$

u

5)

$$-(x^2+4x+\underline{4}-4)$$

$$-((x+2)^2-2^2)$$

$$2^2-(x+2)^2$$

$$\int \frac{1}{\sqrt{2^2-(x+2)^2}}\,dx$$

7) $\int \frac{(2x+6)-6}{x^2+6x+13}\,dx$ $\quad u = x^2+6x+13$
$\quad du = 2x+6$

$\int \frac{2x+6}{x^2+6x+13}\,dx - 6\int \frac{1}{x^2+6x+13-9+9}\,dx$

$\ln|x^2+6x+13| - 6\int \frac{1}{(x+3)^2+4}\,dx$

9) let $u = x^2+1$
$du = 2x\,dx$ $\quad \frac{1}{2}\int \frac{2x}{(x^2+1)^2+1^2}\,dx$

HW: p. 506, # 1 - 9 odd,

13, 19, 21, 25